Basic: What is a function?

A **function** is an object which assigns exactly one output to every input.

- The **domain** of a function is the set of all values which can serve as its input.
- The **range** of a function is the set of all values which can serve as its output.
- Usually we will use letters such as f, g, and h to denote functions. An expression like $f(x)$ represents the output of the function when the input is x.
- Since $f(x) = x^2$ and $g(t) = t^2$ both square the input to get the output, the functions are identical. In other words, $f = g$.
- We may also use informal function notation $y = x^2$ instead of $f(x) = x^2$

Complete the table using formal function notation, informal function notation, and a description.

Prob. #	Formal Function	Informal Function	Description
1	$f(x) = x + 4$		
2		$y = 2x$	
3		$y = x^2 - 3$	
4			Subtract 3 from the input, then multiply by 50
5	$g(x) = 3^x$		
6	$h(t) = t^2 + 2t$		
7		$y = |x|$	
8	$g(x) = \sqrt{x}$		
9		$y = 80$	
10			The negative reciprocal of the input.

Intermediate: Combining Functions

Functions can be combined in many of the same ways that numbers can. For example, the function $f + g$ represents the function whose output is $f(x) + g(x)$ when the input of the function is x.

In other words, $(f + g)(x) = f(x) + g(x)$.

An additional way of combining functions is **function composition**. The function $f \circ g$ (read as "f composed with g") is defined by $(f \circ g)(x) = f(g(x))$. The output of the inner function is used as the input for the outer function.

Let f, g, and h be the functions defined by $f(x) = x + 2$, $g(t) = 2t$, and $h(\theta) = \theta^2$.

Write the formal function rule for each of the following functions.

Prob. #	Function Name	Function Rule
11	$f + g$	
12	$h - g$	
13	$g \cdot h$	
14	$3f + 2g$	
15	$f \circ h$	
16	$h \circ f$	

Advanced: Can a function's input and output be more than just a number?

Functions can have a domain and range beyond just the set of real numbers.

17. Let f be a complex function defined by $f(z) = z^3$. Find $f(2 + i)$.

18. Let d be a function on ordered pairs of real numbers defined by $d\big((x, y)\big) = \sqrt{x^2 + y^2}$. Notice this is equivalent to making d a function of 2 variables. Find $d\big((5, 12)\big)$.

19. Let α be a function on functions defined by $\alpha(f, g) = f + 3g$. If $f(x) = 5x$ and $g(x) = x^2$, find $\alpha(f, g)$.